7

Deductive logic

The preceding five chapters are directed mainly at this book's purpose of cultivating a humanities-rich perspective on science. This is the first of five chapters directed mainly at this book's other purpose of increasing scientific productivity.

Logic is the science of correct reasoning and proof, distinguishing good reasoning from bad. Logic addresses the relationship between premises and conclusions, including the bearing of evidence on hypotheses.

In the context of logic, an "argument" is not a dispute but rather is a structured set of statements in which some statements, the premises, are offered to support or prove others, the conclusions. Many deductive systems, including arithmetic and geometry, are developed on a foundation of logic in the modern and unified vision of mathematics.

Of course, given the simple premises that "All men are mortal" and "Socrates is a man," one trusts scientists to reach the valid conclusion that "Socrates is mortal," even without formal study of logic. But given the more difficult problems that continually arise in science, the rate of logical blunders can increase substantially in the absence of elementary training in logic. Fortunately, most blunders involve a small number of common logical fallacies, so even a little training in logic can produce a remarkable improvement in reasoning skills.

The aim of this chapter differs from that of an ordinary text or course on logic. One short chapter cannot teach logic comprehensively. What it can do, however, is convey an insightful general impression of the nature and structure of deductive logic. Recall that the PEL model introduced in Chapter 5 identifies logic as one of the three essential inputs (along with presuppositions and evidence) required to support scientific conclusions. Consequently, the credibility of science depends on having a logic that is coherent and suitable for investigating the physical world.

This chapter distinguishes the two basic kinds of logic: deductive logic, explained in this chapter, and inductive logic, explored in Chapter 9. One branch of deduction, probability theory, is deferred to the next chapter. The history of logic is reviewed briefly, followed by basic accounts of propositional logic, predicate logic, and arithmetic. Common logical fallacies are analyzed to refine reasoning skills.

Deduction and induction

The distinction between deduction and induction can be explained in terms of three interrelated differences. Of these three differences, the one listed first is the fundamental difference, with the others being consequences or elaborations. Custom dictates distinct appellative terms for good deductive and inductive arguments. A deductive argument is valid if the truth of its premises guarantees the truth of its conclusions and is invalid otherwise. An inductive argument is strong if its premises support the truth of its conclusions to a considerable degree and is weak otherwise. The following deductive and inductive arguments, based on Salmon (1984:14), illustrate the three differences.

Valid Deductive Argument
Premise 1. Every mammal has a heart.
Premise 2. Every horse is a mammal.
Conclusion. Every horse has a heart.

Strong Inductive Argument
Premise 1. Every horse that has been observed has had a heart.
Conclusion. Every horse has a heart.

First, the conclusion of a deductive argument is already contained, usually implicitly, in its premises, whereas the conclusion of an inductive argument goes beyond the information present, even implicitly, in its premises. The technical terms for this difference are that deduction is nonampliative but induction is ampliative. For example, the conclusion of the foregoing deductive argument simply states explicitly, or reformulates, the information already given in its premises. All mammals have hearts according to the first premise, and that includes all horses according to the second premise, so the conclusion follows that every horse has a heart. On the other hand, the conclusion of the foregoing inductive argument contains more information than its premise. The premise refers to some group of horses that have been observed up to the present, whereas the conclusion refers to all horses, observed or not, and past or present or future.

Note that this difference, between ampliative and nonampliative arguments, concerns the relationship between an argument's premises and conclusions, specifically whether or not the conclusions contain more information than the premises. This difference does not pertain to the conclusions as such, considered in isolation from the premises – indeed, the foregoing two arguments have exactly the same conclusion.

Second, given the truth of all of its premises, the conclusion of a valid deductive argument is true with certainty, whereas even given the truth of all of its premises, the conclusion of an inductive argument is true with at most high probability. This greater certainty of deduction is a direct consequence of its being nonampliative: "The [deductive] conclusion must be true if the premises are true, *because* the conclusion says nothing that was not already stated by the premises" (Salmon 1984:15). The only way that the conclusion of a valid deductive argument can be false is for at least one of its premises to be false. On the other hand, the uncertainty of induction is a consequence of its being ampliative: "It is because the [inductive] conclusion says something not given in the premise that the conclusion might be false even though the premise is true. The additional content of the conclusion might be false, rendering the conclusion as a whole false" (Salmon 1984:15). For example, the foregoing inductive conclusion could be false if some other horse, not among those already observed and mentioned in this argument's premise, were being used for veterinary research and had a mechanical pump rather than a horse heart.

Deductive arguments are either valid or invalid on an all-or-nothing basis because validity does not admit of degrees. But inductive arguments admit of degrees of strength. One inductive argument might support its conclusion with a very high probability, whereas another might be rather weak.

The contrast between deduction's certainties and induction's probabilities can easily be overdrawn, however, as if to imply that induction is second-rate logic compared with deduction. Representing certain truth by a probability of 1 and certain falsehood by 0, an inductive conclusion can have any probability from 0 to 1, including values arbitrarily close to 1 representing certainty of truth (or 0 representing certainty of falsehood). Given abundant evidence, induction can deliver practical certainties, although it cannot deliver absolute certainties.

Third and finally, deduction typically reasons from the general to the specific, whereas induction reasons in the opposite direction, from specific cases to general conclusions. That distinction was prominent in Aristotle's view of scientific method (Losee 2001:5–8) and remains prominent in today's dictionary definitions. For instance, the *Oxford English Dictionary* defines "deduction" as "inference by reasoning from generals to particulars," and it defines "induction" as "The process of inferring a general law or principle from the observation of particular instances." Deduction reasons from a given model to expected data, whereas induction reasons from actual data to an inferred model, as depicted in Figure 7.1.

As encountered in typical scientific reasoning, the "generals" and "particulars" of deduction and induction have different natures and locations. The general models or theories exist in a scientist's mind, whereas the particular instances pertain to physical objects or events that have been observed. Often,

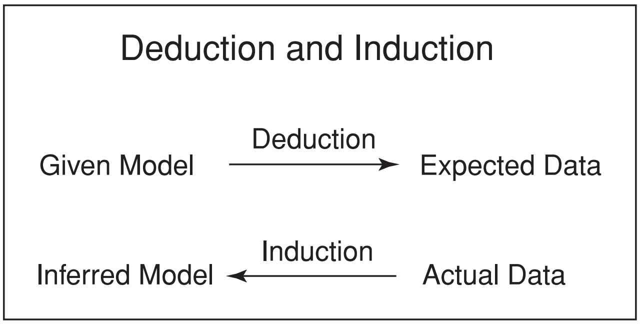

Figure 7.1 The opposite reasoning directions of deduction and induction. Deduction reasons from the mind to the world, whereas induction reasons from the world to the mind.

the observations or data comprise a limited sample, but the researchers are interested in the larger population from which the sample was drawn. For instance, a clinical trial may examine a representative sample of persons to reach conclusions pertaining to the whole population of persons suffering from a given disease.

Deduction is neither better than induction nor worse. Rather, they pursue answers to different kinds of questions, with deduction reasoning from a mental model to expected data, and induction reasoning from actual data to a mental model. Both are indispensable for science.

Historical perspective on deduction

Aristotle (384–322 BC) wrote extensively on logic. Although his works on some topics, including natural science, suffered much neglect until the early 1200s, his corpus on logic, the *Organon* or tool (of reasoning), fared better. Aristotle's logic built on ideas from Socrates and Plato. Epicurean, Stoic, and Pythagorean philosophers also developed logic and mathematics. Besides Greece, there were impressive ancient traditions in logic in Babylon, Egypt, India, and China. Largely because of Augustine's early influence, the Aristotelian tradition came to dominate logic in the West, so that tradition is emphasized here.

In his *Prior Analytics*, Aristotle taught that every belief comes through either deduction or induction. His syllogistic logic is the first deductive system, predating Euclid's geometry. Aristotle proposed an inductive–deductive model of scientific method that features alternation of deductive and inductive steps. This alternation, moving from mental model to physical world and back again, leads scientists to a mind–world correspondence – to truth. In this process, any discrepancy between model and world is to be resolved by adjusting the model to the world because the actual data in the inductive step have priority over the expected data in the deductive step. Assuming that the data are not faulty or excessively inaccurate, actual data contrary to a model's expectations imply that something is wrong with the model. Adjusting one's model to the world is the basis of scientific realism.

Euclid (*fl. c.* 300 BC) was the great master of geometry. Many truths of geometry were known before Euclid. For example, earlier Babylonians and Egyptians knew that the sum of the interior angles of a triangle equals 180 degrees. But that was known by empirical observation of numerous triangles, followed by inductive generalization. Their version of geometry was a practical art related to surveying, in line with the name "geometry," literally meaning "earth measure."

Euclid's *Elements of Geometry*, in one of the greatest paradigm shifts ever, instead demonstrated those geometrical truths by deduction from several axioms and rules. Euclid's geometry had five postulates concerning geometry, such as that a straight line can be extended in either direction, plus five axioms or "common notions" concerning correct thinking and mathematics in general, such as that the whole is greater than its parts. Euclid's combination of geometrical postulates and logical axioms represented a nascent recognition that logic underlies geometry. Countless theorems can be deduced from Euclid's postulates and axioms, including that the sum of the interior angles of a triangle equals 180 degrees.

Subsequently, non-Euclidean geometries were discovered by Thomas Reid (1710–1796), Nikolai Lobachevsky (1792–1856), János Bolyai (1802–1860), Bernhard Riemann (1826–1866), and others. This rendered Euclid's work *a* geometry rather than *the* geometry. In Reid's alternative geometry, the sum of the interior angles of a triangle equals more than 180 degrees.

Anicius Manlius Severinus Boethius (AD 480–524) translated, from Greek into Latin, many parts of Aristotle's logical works, Porphyry's *Introduction to Aristotle's Logic*, and parts of Euclid's *Elements*. His *On Arithmetic*, based on earlier work by Nicomachus of Gerasa, became the standard text on arithmetic for almost a millennium.

Peter Abelard (*c.* 1079–1142) wrote four books on logic. He and his students, John of Salisbury and Peter Lombard, greatly influenced medieval logic. The use of Arabic numerals was spread into Europe by Alexandre de Villedieu (*fl. c.* 1225), a French Franciscan, John of Halifax (or Sacrobosco, *c.* 1200–1256), an English schoolman, and Leonardo of Pisa (or Fibonacci, *c.* 1180–1250), an Italian mathematician. The modern mind can hardly imagine the tedium of multiplication or division using Roman numerals, or how few persons in medieval Europe could perform what we now regard as elementary calculations.

Albertus Magnus (*c.* 1200–1280) wrote 8,000 pages of commentary on Aristotle, including much logic. He also wrote a commentary on Euclid's *Elements*.

Robert Grosseteste (*c.* 1168–1253) founded the mathematical-scientific tradition at Oxford. He affirmed and refined Aristotle's inductive–deductive scientific method, which he termed the "Method of Resolution and Composition" for its inductive and deductive components, respectively. Also, his Method of Verification involved deriving the deductive consequences of a theory beyond the original facts on which the theory was based and then observing the actual outcome in a controlled experiment to check the theory's predictions. That method recognized the priority of data over theories, in accord with Aristotle. Grosseteste's Method of Falsification eliminated bad theories or explanations by showing that they imply things known to be false. To increase the chances of eliminating false theories, he recommended that conclusions reached by induction be submitted to the test of further observation or experimentation.

Putting all those methods together, the objective of Grosseteste's new science was to make theory bear on the world and the world bear on theory, thereby bringing theory into correspondence with the world. Grosseteste's scientific method sought to falsify and reject false theories, to confirm and accept true theories, and to discern which kinds of observational or experimental data would help the most in theory evaluation.

There is substantial similarity between Grosseteste's medieval science and modern science. "Modern science owes most of its success to the use of these inductive and experimental procedures, constituting what is often called 'the experimental method'. The . . . modern, systematic understanding of at least the qualitative aspects of this method was created by the philosophers of the West in the thirteenth century. It was they who transformed the Greek geometrical method into the experimental science of the modern world" (Crombie 1962:1). I concur with this assessment that a basically correct and complete scientific method emerged in the thirteenth century.

William of Ockham (*c.* 1285–1347) wrote a substantial logic text, the *Summa logicae*. The principle of parsimony is often called Ockham's razor because of his influential emphasis on this principle. Jean Buridan (*c.* 1295–1358) wrote the *Summulae de dialectica*, a then-modern revision and amplification of the earlier logic text by Peter of Spain (*fl.* first half of the thirteenth century), and two advanced texts, the *Consequentiae* and *Sophismata*.

René Descartes (1596–1650) was the founder of analytic geometry. Blaise Pascal (1623–1662) contributed to projective geometry, arithmetic, combinatorial analysis, probability, and the theory of indivisibles (a forerunner of integral calculus). He developed the first commercial calculating machine. Isaac Newton (1642–1727) and Gottfried Leibniz (1646–1716) invented calculus. Giuseppe Peano (1858–1932) devised axioms for arithmetic.

For millennia, the various branches of deduction – such as logic, arithmetic, and geometry – had been developed as separate and unrelated systems. Early great works aiming to unify logic and mathematics were the brilliant *Grundgesetze der Arithmetik* (*The Basic Laws of Arithmetic*) of Frege (1893) and the monumental *Principia Mathematica* of Whitehead and Russell (1910–13).

Table 7.1. Truth-table definitions for negation, conjunction, disjunction, implication, and equality

Assignments		Not	And	Or	Implies	Equals
A	B	$\sim B$	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \equiv B$
T	T	F	T	T	T	T
T	F	T	F	T	F	F
F	T		F	T	T	F
F	F		F	F	T	T

Propositional logic

Propositional logic, also called statement calculus and truth-functional logic, is a rather elementary branch of deductive logic. Nevertheless, it is quite important because it pervades common-sense reasoning and scientific reasoning.

A simple proposition has a subject and a predicate, such as "This apple is red" or "Mary is coming." Propositional logic considers only declarative statements. Accordingly, every simple proposition has the property of having one or the other of two possible truth-values: true (T) and false (F). Note that the truth-value applies to the proposition as a whole, such as "This apple is red" being true for a red apple but false for a green apple. In propositional logic, as introduced in this section, there is no further analysis of the subject and predicate within a proposition. But, in predicate logic, to be explained in the next section, further analysis is undertaken. Hence, predicate logic is more complicated, subsuming propositional logic and adding new concepts and analysis.

Proposition constants represent specific simple propositions and are denoted here by uppercase letters like A, B, and C (except that T and F are reserved to represent the truth-values true and false). For example, "The barometer falls" can be symbolized by B, "It will rain" by R, and "It will snow" by S. Then, the compound sentence "If the barometer falls, then either it will rain or it will snow" can be expressed by "If B, then R or S."

The most common connectives or operators are "not," "and," "or," "implies," and "equals." They are also termed negation, conjunction, disjunction, implication, and equality. These five connectives are denoted here by these symbols: "$\sim$," "$\wedge$," "$\vee$," "$\rightarrow$," and "$\equiv$." The meanings of these connectives are specified by a truth table (Table 7.1).

"Not" is a unary operator applied to a single proposition. If B is true, then $\sim B$ is false; and if B is false, then $\sim B$ is true. That is, B and $\sim B$ have opposite truth-values. The other connectives are binary operators applied to two propositions. For example, "A and B," also written as "$A \wedge B$," is true when both A is true and B is true and is false otherwise. Simple propositions can be combined with connectives, such as $B \to (R \vee S)$ to symbolize the preceding compound proposition about a barometer. Parentheses are added as needed to avoid ambiguity. To simplify expressions, the conventions are adopted that negation has priority over other connectives and applies to the shortest possible sub-expression, and parentheses may be omitted whenever the order makes no difference.

Incidentally, two other logical operators, not already specified in Table 7.1, are important in computer design because they can be implemented with simple transistor circuits. Joint denial of A and B, expressed by "Neither A nor B" and symbolized by "$A \downarrow B$," equals the negation of "A or B" and hence is also named "Nor." An alternative denial of A and B, expressed by "Either not A or not B" and symbolized by "$A \mid B$," equals the negation of "A and B" and hence is also named "Nand." (To avoid potential confusion, note that this symbol "|" instead means "or" in several computer-programming languages.) Remarkably, all of the logical operators in Table 7.1 can be defined or replaced by joint denial alone, or by alternative denial alone. For instance, $\sim A$ is logically equivalent to $A \downarrow A$ or to $A \mid A$. Likewise, $A \wedge B$ is logically equivalent to $(A \downarrow A) \downarrow (B \downarrow B)$ or to $(A \mid B) \mid (A \mid B)$. Consequently, circuits using Nor and Nand operations are extremely useful in computers. Annually, the world produces more transistors than it produces grains of wheat or grains of rice.

Proposition variables stand for simple propositions and are denoted here by lowercase letters like p and q. Hence, the variable p could stand for the constant A or B or C. Proposition expressions are denoted here by script letters and are formed by one or more applications of two rules: (1) any proposition constant or variable is a proposition expression; and (2) if $\mathcal{A}$ and $\mathcal{B}$ are proposition expressions, then their negations are proposition expressions as well as their being combined by conjunction, disjunction, implication, and equality.

An argument is a structured, finite sequence of proposition expressions, with the last being the conclusion (ordinarily prefaced by the word "therefore" or the symbol "$\therefore$"), and the others the premises. The premises are intended to support or prove the conclusion. For example, *modus ponens* is a valid argument with two premises and one conclusion: A; A implies B; therefore B. Likewise, *modus tollens* is the valid argument: not B; A implies B; therefore not A. Incidentally, the full Latin names are *modus ponendo ponens* meaning "the way that affirms by affirming," and *modus tollendo tollens* meaning "the way that denies by denying." An argument is valid if under every assignment of truth-values to the proposition variables that makes all premises true, the conclusion is also true. Otherwise, the argument is invalid.

There are several methods for proving that an argument is valid or else invalid, as the case may be. Different methods all give the same verdict, but one

Formal Propositional Logic

(1) Language. The symbols used are as follows: ~, →, (,), p_1, p_2, p_3, and so on.
(2) Expressions. A well-formed formula (wff) is formed by one or more applications of two rules. (a) Each p_i is a wff. (b) If A and B are wffs, then $(\sim A)$ and $(A \rightarrow B)$ are wffs.
(3) Axioms. For any wffs A, B, and C, axioms are formed by the following three axiom schemes:
Axiom Scheme 1. $(A \rightarrow (B \rightarrow A))$
Axiom Scheme 2. $((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)))$
Axiom Scheme 3. $(((\sim A) \rightarrow (\sim B)) \rightarrow (B \rightarrow A))$
(4) Rule. The rule, *modus ponens*, says from A and $(A \rightarrow B)$, infer B.
(5) Interpretation. The symbols "~" and "→" are the logical connectives negation and implication, which may have associated parentheses needed to specify the order of operations, and the symbols p_1, p_2, p_3, and so on, represent proposition variables having truth-values of either true or false. A proof is a sequence of wffs $A_1, \ldots, A_N, A$ such that each wff either is an axiom or follows from two previous members of the sequence by application of *modus ponens*. The final wff, A, is a theorem.

Figure 7.2 The elements of formal propositional logic. This logic is specified by its language, expressions, axioms, rule, and interpretation.

method may be easier to understand or use in a given instance than is another. The conceptually simplest method, directly reflecting the definitions of validity and invalidity, is to construct a truth table to determine whether or not each assignment of truth-values to the argument's proposition variables that makes all premises true also makes the conclusion true. Another method for proving validity is to deduce the argument as a theorem from the axioms and rules. But proof strategies are best left to any standard logic text.

Figure 7.2 presents a formal system for propositional logic, drawing on Hamilton (1978:28). Some liberty has been taken to simplify this presentation. The ordinary letters "*A*," "*B*," and "*C*" in this figure should actually be script letters to represent proposition expressions, not merely proposition constants as denoted by these ordinary letters elsewhere in this section.

Propositional logic is both sound and complete. Basically, this means that its rules are correct and that no additional rules are needed. Propositional logic is also decidable, meaning that any argument can be proven to be valid or else invalid. For example, consider the argument: $A \rightarrow B$; $B \rightarrow (C \vee D)$; $A \wedge \sim C$; $\therefore D$. Is it valid or invalid? Some time and effort are required to render the verdict, which turns out to be that this argument is valid. But before even starting to assess validity, it is already known and guaranteed in advance that the outcome is predetermined by the axioms of propositional logic and the answer is decidable.

Predicate logic

Predicate logic, also called first-order logic and predicate calculus, subsumes and surpasses prepositional logic. It adds two extensions. First, it distinguishes between a proposition's subject and predicate. In a conventional symbolism, the predicate is denoted by an uppercase letter and the subject is denoted by the following lowercase letter placed within parentheses. For instance, "This apple is red," with its predicate "is red" and subject "This apple," can be symbolized by $R(a)$. Second, predicate logic has the existential quantifier "some" denoted by "∃" and the universal quantifier "all" denoted by "∀." For example, $(\forall x)(A(x))$ means "All x are A" and $(\exists x)(A(x))$ means "Some x is A."

One kind or subset of predicate logic is syllogistic logic. A familiar example is the argument: All men are mortal; Socrates is a man; therefore, Socrates is mortal. Because of its deeper analysis distinguishing subjects and predicates and its inclusion of existential quantifiers such as "all," predicate logic can analyze this argument and declare this syllogism valid. But the simpler propositional logic cannot express or handle syllogisms.

A formal deductive system for predicate logic is about twice as complicated as the one shown in Figure 7.2 for propositional logic (Hamilton 1978:49–56, 71–72). By contrast, the range of theorems that predicate logic can prove is incomparably greater than the range for propositional logic. Accordingly, predicate logic supplies the powerful logic that lays the foundation upon which other branches of mathematics can be constructed, including arithmetic and probability.

Arithmetic

In the contemporary vision of deductive systems, numerous branches of mathematics, such as arithmetic, are all built on a foundation of predicate logic. To build a branch of mathematics on logic, two items must be added: an interpretation and some axioms. A formal language is abstract, and an interpretation attaches a particular meaning to some symbols of a formal language, such as arithmetic being about numbers. Additional axioms are needed because often a mathematical statement is true (or false) because of the mathematical meanings of its terms, rather than merely the logical arrangement of its terms. There are both logical truths and mathematical truths. Both require axioms.

Although syllogistic logic was axiomatized by Aristotle and geometry by Euclid more than two millennia ago, arithmetic was axiomatized only just over a century ago in 1889 by Giuseppe Peano. His axioms can be edited in various ways to make them somewhat more transparent or convenient. Figure 7.3 presents a formulation with nine axioms.

Peano Axioms for Arithmetic

1. 0 is a natural number.
2. For every natural number x, $x = x$.
3. For all natural numbers x and y, if $x = y$, then $y = x$.
4. For all natural numbers x, y, and z, if $x = y$ and $y = z$, then $x = z$.
5. For all a and b, if a is a natural number and $a = b$, then b is also a natural number.
6. For every natural number n, its successor $S(n)$ is a natural number.
7. For every natural number n, $S(n) = 0$ is false.
8. For all natural numbers m and n, if $S(m) = S(n)$, then $m = n$.
9. If K is a set such that (a) 0 is in K and (b) for every natural number n, if n is in K, then $S(n)$ is in K; then K contains every natural number.

Figure 7.3 The nine Peano axioms for arithmetic. The first axiom assumes that zero is a natural number, the next four describe the equality relation, the next three describe the successor function (where 1 is the successor of 0, 2 is the successor of 1, and so on), and the last axiom concerns the set of all natural numbers.

Arithmetic can be developed from the Peano axioms and the inherited predicate logic. For the most part, the meanings of the Peano axioms should be fairly obvious. For instance, axiom 2 says that every number equals itself, and axiom 8 says that if $m + 1 = n + 1$, then $m = n$.

To reiterate an important point about predicate logic from the preceding section in the present context of arithmetic, axioms fix in advance the outcome for subsequent theorems or calculations. For example, is $871 \times 592 = 515432$ correct? A little effort is required to check this calculation, but before even starting, the verdict has been predetermined by the arithmetic axioms. Actually, this calculation is incorrect, the proper value being 515632. Precisely because the rules of arithmetic are fixed before the game begins, arithmetic is meaningful and rational. If different persons could get different sums for $27 + 62$, then in such a world there would be no science, and no banks either.

Many persons may miss the wonder, but Albert Einstein asked "How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?" (Frank 1957:85). Likewise, Potter (2000:17–18) expressed this wonder specifically as regards arithmetic, remarking that "it is not immediately clear why the properties of abstract objects [numbers] should be relevant to counting physical or mental ones. . . . One has only to reflect on it to realize that this link between experience, language, thought, and the world, which is at the very centre of what it is to be human, is truly remarkable."

Indeed, there is something wonderful about arithmetic's effectiveness. It may be noted, however, that the Peano axioms generate standard arithmetic, whereas there also exist equally internally coherent but different *non*standard arithmetics. For instance, in standard (Peano) arithmetic $2 + 2 = 4$. But, in the nonstandard ring arithmetic based on the circular and repeating arrangement of integers 0, 1, 2, 3, 0, 1, 2, 3, 0, and so on, the sum of interest becomes $2 + 2 = 0$. Likewise, in the ring arithmetic with just 0, 1, and 2 repeating, the sum becomes $2 + 2 = 1$. All three of these arithmetics are equally internally coherent, although they are also different from each other.

There are occasional practical uses for nonstandard arithmetics or geometries. For example, standard arithmetic says that $11 + 3 = 14$. But an ordinary clock is based on a circular arrangement of its integers from 1 to 12, so this ring arithmetic says that 3 hours after 11 o'clock, the time is 2 o'clock, or $11 + 3 = 2$. (Or, some clocks have instead the integers 1 to 24 written in a circle). For another example, ordinary surveying or earth-measure uses ordinary geometry. But airplane pilots traveling great distances use the non-Euclidean geometry that Reid invented (for studying optics in a roughly spherical eye) to follow the shortest great circle bearing on our spherical earth, thereby saving time and fuel.

But apart from these understandable exceptions, standard logic and arithmetic and geometry prevail in daily life. While properly appreciating the wonder of arithmetic, part of the reason that (standard) arithmetic fits with our experiences in the physical world is that the choice of standard over nonstandard arithmetic has been guided preemptively by our interests and needs as incarnate beings living in the physical world. That is, in choosing an arithmetic (or geometry or whatever), coherence is not our only criterion but also fit with our experiences of the world. Hence, in the mathematical world of coherent arithmetics, one can obtain $2 + 2 = 0$ or $2 + 2 = 1$ or $2 + 2 = 4$. However, in the physical world of actual objects and events, standard arithmetic is uniquely appropriate. Two apples plus two apples equals four apples.

Common fallacies

Ever since Aristotle's *Sophistical Refutations*, logicians have been providing helpful analyses and classifications of logical fallacies. Furthermore, science educators report that "all the standard logical fallacies, known since Aristotle's day, are routinely committed by science students" (Matthews 2000:331).

There are many fine books and resources on fallacies. But the book by Madsen Pirie, with its generous list of 79 fallacies, is outstanding because of its fun rhetoric in the guise of a naughty sophist. He explained: "This book is intended as a practical guide for those who wish to win arguments. It also teaches how to perpetrate fallacies with mischief at heart and malice aforethought. . . . I have given the reader recommendations on how and where the fallacy may be used to deceive with maximum effect. . . . In the hands of the wrong person this is more of a weapon than a book, and it was written with that wrong person in

Figure 7.4 The logical fallacy *argumentum ad lapidem*, argument to a stone. Samuel Johnson vigorously kicks a stone, attempting to refute the idea that the physical world does not exist, while an unimpressed George Berkeley observes. (This drawing by Carl R. Whittaker is reproduced with his kind permission.)

mind" (Pirie 2006:ix–x). This is the book that everyone needs as we set about the all-important business of getting our own way!

The study of fallacies best begins with its opposite, the study of right thinking. Knowing the genuine article makes its counterfeits more obvious. Recalling the PEL model in Chapter 5, the essence of scientific thinking is evidence that is *admissible* relative to the presuppositions and *relevant* relative to the hypotheses, as well as deductive and inductive *logic* to draw conclusions and weigh evidence. The three italicized words emphasize the principal opportunities for defects: inadmissible evidence, irrelevant evidence, and fallacious logic. The fourth and final category of fallacies reviewed in this section involves a personal rather than a procedural defect, failure of will to pursue the truth.

Inadmissible Evidence. The *argumentum ad lapidem* (argument to a stone) appeals to inadmissible evidence. This fallacy is named for a famous incident depicted in Figure 7.4. George Berkeley had argued that only minds and ideas exist, not physical objects and events, as mentioned in Chapter 5. When Dr. Samuel Johnson was told by James Boswell that this argument is impossible to refute, he vigorously kicked a stone, exclaiming “I refute it thus.”

But as Pirie (2006:101–104) observed, Johnson was not so much refuting Berkeley’s argument as ignoring it. Johnson was presuming a realist interpretation or ontology regarding the empirical evidence provided by sight or feel or sound or any other sense, which is precisely the matter in dispute given Berkeley’s idealist ontology. As emphasized in Chapter 5, the existence and comprehensibility of the physical world are presuppositions of mainstream science, not conclusions of science (or philosophy either). To think otherwise is to commit the *argumentum ad lapidem* fallacy. Berkeley could accept that Johnson had an experience of kicking a stone and could even share that experience with him. But Berkeley would not infer from this experience the metaphysical theory that the stone has an independent physical existence. Presuppositions cut deeper than evidence.

Irrelevant Evidence. Several fallacies appeal to evidence that is admissible, given the common-sense presuppositions of mainstream science, but that evidence is irrelevant because it fails to bear on the credibilities of the various hypotheses under consideration. One such fallacy is the *argumentum ad hominem* (argument to the man), which attacks the person promoting the disliked idea rather than the idea itself. For instance, a theory could be attacked by saying its proponent is a teacher at a small community college.

Another fallacy is the *red herring*. This draws attention away from the original argument to some other matter that is irrelevant but provides an easier target for refutation.

An alluring fallacy for scientists is *unobtainable perfection*, or at least excessive perfection. This fallacy discredits a result by requiring greater accuracy or scope. For instance, if a paper under review compares methods *A* and *B*, a reviewer might say that it must also compare method *C* in order to be publishable. But simply to complain that more could be done is irrelevant because this is always the case. Rather, the relevant criteria are whether that paper adds to what was known before and whether it has some theoretical interest or practical value. Also, adding method *C* may be a good idea, but the editor might intervene and propose this as a suggestion or recommendation rather than a requirement.

Fallacious Logic. Most logical fallacies obtain their apparent plausibility from being subtle variations on other arguments that are valid. Logical fallacies are especially deceptive when their conclusions are already believed or desired.

Fallacies result from invalid variations on the valid argument *modus ponens*: *A*; *A* implies *B*; therefore *B*. The implication “*A* implies *B*” consists of the antecedent *A* and the consequent *B*. Hence, the valid argument *modus ponens* affirms the antecedent. Similarly, the valid argument *modus tollens* denies the consequent: not *B*; *A* implies *B*; therefore not *A*. But other variations are invalid. Affirming the consequent is the logical fallacy: *B*; *A* implies *B*; therefore *A*.

An example is: The plants are yellowish; if plants lack nitrogen, then they become yellowish; therefore the plants lack nitrogen. Likewise, denying the antecedent is also invalid: not *A*; *A* implies *B*; therefore not *B*. A common version of this fallacy is the argument from *missing evidence*. However, an observation of missing evidence *A* has no force in rejecting theory *B* unless it is supplemented with an additional argument showing that evidence *A* would be expected to exist, and perhaps even to be abundant, were theory *B* true. Furthermore, an honest evaluation of theory *B* would also consider whether some other kinds of evidence are relevant and available rather than just eagerly pursuing the easiest possible way to discredit *B*.

Syllogisms have 256 possible forms, of which only 24 are valid. An example of a valid syllogism is: Socrates is a man; all men are mortal; therefore Socrates is mortal. Because most forms are invalid, apart from some training in logic, syllogisms offer numerous opportunities for tricky fallacies.

A *false dilemma* mentions fewer alternatives than actually exist. In the false dilemma "*A* or else *B*; not *A*; therefore *B*," the logical form is valid, but the first premise "*A* or else *B*" is false because of additional possibilities such as *C*. For example, "Either apply nitrogen fertilizer or get yellowish plants" is a false dilemma for many reasons, including the possibilities that a particular soil already has adequate nitrogen without adding fertilizer, or that a virus causes yellowish plants despite adequate fertilizer. Of course, the opposite fallacy also occurs: the "optionitis" of believing that one has more options than reality (or feasibility) actually offers. Some dilemmas are real.

A variant on the false dilemma is the *straw-man argument*. The logical form is this same "*A* or else *B*; not *A*; therefore *B*," where *A* represents an opponent's position and *B* the favored position. However, the premise "not *A*" is supported by attacking the opponent's weakest evidence or a simplistic misrepresentation of the opponent's position. An honest refutation of the opponent's position must instead represent *A* accurately and tackle its strongest evidence and arguments.

Yet another variant on the false dilemma is the *argumentum ad ignorantiam* (argument from ignorance). This fallacy attempts to drive opponents to accept an argument unless they can find a better argument to the contrary. For example, an environmentalist might say "We cannot prove that this pesticide is safe, so we must assume that it is dangerous and outlaw its use." There may or may not be some other good arguments against this pesticide's safety, but an argument from ignorance is not a good reason. The implicit dilemma in an appeal to ignorance is "Give me a better argument, or else accept my argument." But the unmentioned third option is to admit current inability to construct a better argument while still either rejecting the offered argument or suspending judgment.

Failure of Will. Given the dishonorable nature of failure of will, this fourth and final category of fallacies is best discussed by adopting Pirie's guise as a naughty sophist. After all, adroit evasion of knowledge, while still giving every appearance of energetic pursuit of knowledge, requires considerable skill!

Three fallacies are useful to conceal failure of will: privileged cynicism, secret alliance, and personal exemption. Admittedly, these are imperfect means for putting a pretty face on failure of will. But these three fallacies work as well as can be expected, given the inherent challenges of this naughty business. Their principal merits are that these fallacies are unrivaled in their resistance to remediation, and sometimes they can even achieve self-deception!

An effective fallacy for implementing failure of will is *privileged cynicism*: When there is a spectrum of positive to negative opinions about something's merit, the most negative, skeptical, cynical opinion is privileged by being presented and perceived as automatically the view of the sophisticated elite – unlike the naïve and despised view of the ignorant commoners. For instance, the commoners (including most practicing scientists!) may think that much knowledge is readily attained and perfectly solid, whereas presumably the academic elite is steeped in postmodern rejection of knowledge claims, so privileged cynicism declares that *automatically* the latter group has the more sophisticated view. A skilled professor can wield this fallacy to encourage students in a cynical attitude that then becomes the students' passport into the alluring world of the cultural elite. Inside such a culture, cynicism equals sophistication.

This fallacy of privileged cynicism applies readily to science. Students can be lured easily into the mighty gratifying feeling that they, being superior to the gullible commoners, are getting the real, dirty story on what science is. The fallacy of privileged cynicism has great appeal to persons who already feel disappointed or disenfranchised in life for any reason.

A huge advantage of privileged cynicism is its ease. A lackluster high school student, let alone a bright college student, can learn five skeptical or cynical remarks in as many minutes. Furthermore, merely two or three pages suffice for a skilled writer to display a cynical view of science in all its glory, which seems to call for automatic assent from any reader wishing to be numbered among the sophisticates who are in the know. By contrast, a satisfactory account of actual scientific method takes work to write and work to read. Hence, the hard-won sophistication of a working scientist cannot possibly compete with the cynical version of "sophistication" in terms of being offered on the cheap.

A second fallacy for implementing failure of will is *secret alliance*. This wonderfully subtle fallacy involves fighting an intense battle not so much for its own intrinsic importance as for its strategic value in defending an ally in a larger war, while that ally receives so little explicit mention as to remain virtually a secret. Thereby, the real motivations for the battle are not obvious, perhaps even to many of the battle's most prominent combatants on both sides.

The main example in the realm of science is the notorious "science wars" reviewed in Chapter 4. The intensity of this intellectual war, augmented by melodramatic and inflammatory rhetoric, is astonishing and perhaps even mystifying. Why is it so intense? One might suspect that often the underlying motivations have not been expressed in an entirely forthright manner. Indeed, whether a person's intellectual verdict is that the prospects for human knowledge are dim or bright, and whether that person's emotional reaction to this verdict is sad or happy, are two separate matters. Rather than the usual giddy triumph over vanquished truth, why not express instead a crushing sadness over unrelenting ignorance? This love of ignorance and uncertainty demands some explanation!

Occasionally, there are revealing remarks that arouse suspicions of a secret alliance, although that alliance may be subtle enough to operate at an unconscious level. For example, philosopher Brown (1987:230) remarked, "I have offered here one detailed argument for the now familiar thesis that there is no fundamental methodological difference between philosophy and science. . . . [But] it has become progressively clearer that the sciences cannot provide certainty and have no a priori foundation. . . . [Admittedly,] earlier thinkers believed that both science and philosophy provide certain knowledge of necessary truths. We must conclude that neither do. . . . [The] human intellect . . . seems unable to grasp a final truth." So, chastened science has no truth, and now philosophy can enjoy the same!

For another example, science educator Meyling (1997) mentioned one of his high school students who began with a common-sense, realistic view of science, but in the end she accepted her teacher's "fallibilistic-pluralistic model of epistemology" of "existential uncertainty" and "the tentativeness of science." Meyling quoted her saying that "Truth is relative, we have to get used to that, there are only things that are more correct than others, but there is nothing that is absolutely correct. . . . When you think you know the truth, you force others to think and live that way. . . . This is a claim on absoluteness that cannot be justified – by no one and by no theory." Meyling commented that "I believe that this recognition is far more important than the knowledge about a whole set of scientific 'facts'," and he was particularly pleased that his student extended her new skeptical epistemology to the "ethical level." He mentioned a letter he had received, in which "Sir Karl R. Popper was very pleased with this quote." But Meyling's enthusiasm and Sir Karl's praise notwithstanding, some parents may feel that a science classroom is not a fitting place to encourage ethical relativism or skepticism in other persons' children.

The rather popular idea that science is the sole source and guardian of empirical evidence, and hence of all objective and public knowledge, is a mistake that can seemingly justify failure of will in other realms outside science. But this mistake cannot be supported by mainstream science, which maintains the exact opposite: that scientific thinking, with public evidence as its foremost feature, is also applicable beyond science itself in the humanities and everyday life. Nor can it be supported by insistence on methodological naturalism because this is a stipulatory convention within natural science that is inherently inappropriate in many other disciplines that also use empirical and public evidence. Nevertheless, this mistaken idea of scientism is easily motivated and long sustained by the most potent of fallacies, failure of will. Frankly, for those persons who heartily want empirical evidence to work for technological comforts *and* not to work for worldview inquiries, simplistic arguments – preferably expressed in a mere sentence or two – should provide welcome and adequate reassurance. On the other hand, for other persons who heartily want empirical evidence to work for technological comforts *and* scientific discoveries *and* worldview inquiries, energetic study of mainstream science and mainstream philosophy should prove fruitful. Getting the most knowledge and benefit from empirical and public evidence requires engaging both the sciences and the humanities, in alignment with the appealing AAAS (1990) vision of science as a liberal art participating in an exciting wider world.

A third and final fallacy for implementing failure of will is *personal exemption.* This fallacy involves mastering fallacies for the purpose of dismantling and evading other persons' arguments, while ignoring the responsibility of detecting and correcting one's own fallacies, as if one has a personal exemption from dealing with truth and reality. The following chapters on probability and statistics examine additional fallacies.

Summary

Logic is the science of correct reasoning and proof. It addresses the relationship between premises and conclusions, including the bearing of evidence on hypotheses. A deductive argument is valid if the truth of its premises entails the truth of its conclusions and is invalid otherwise. Formal deductive logic begins with a language, axioms, and rules and then derives numerous theorems.

As applied in science, deductive logic argues with certainty from an assumed model to particular expected data. By contrast, inductive logic argues with probability from particular actual data to an inferred general model. In its pursuit of realism and truth, scientific thinking alternates deduction, reasoning from mind to world, and induction, reasoning from world to mind.

The first deductive systems to be axiomatized were syllogisms by Aristotle and then geometry by Euclid. Medieval philosopher-scientists advanced deductive logic considerably. Arithmetic was finally axiomatized only just over a century ago by Peano. The modern vision of deduction, which unites all of its branches into a single unified system built on a base of predicate logic, began with stunningly brilliant work by Frege and by Whitehead and Russell.

The formal system for propositional logic presented here has three axioms and one rule. The axioms for predicate logic are about twice as complicated, but the resulting range of theorems that predicate logic can prove is incomparably greater than the range for propositional logic. Peano arithmetic is presented with nine axioms. Probability is another branch of deductive logic, but that topic is deferred to the next chapter.

Fallacies have received much interest since Aristotle. Fallacies are best understood and categorized after first recalling the key resources of scientific thinking: admissible and relevant evidence, and deductive and inductive logic. Accordingly, three major categories of fallacies are inadmissible evidence, irrelevant evidence, and fallacious logic. The fourth and final category of fallacies reviewed in this chapter involves a personal rather than a procedural defect: failure of will to pursue the truth.

Study questions

(1) What are the three interrelated differences between deductive and inductive arguments? Is deduction superior to induction, or are they complementary in scientific thinking?

(2) What are the truth-table definitions for the logical operators Nor and Nand? Why are these operators so extremely useful in computer circuits?

(3) What two main sorts of considerations inform axiom choice for any standard version of a deductive system, such as standard logic or standard arithmetic? What are some applications for nonstandard arithmetic and non-Euclidean geometry?

(4) What is the fallacious *argumentum ad lapidem*, the argument to a stone? Can you contrive an alluring example? How does this fallacy relate to science's presuppositions?

(5) Failure of will to pursue the truth can be implemented by various means, including privileged cynicism, secret alliance, and personal exemption. Give an example of each. Might failure of will be a contributing factor in attacks on science's rationality? Explain your answer.